

Exact black hole solutions in shift-symmetric scalar-tensor theories

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We derive a variety of exact black hole solutions in a subclass of Horndeski's scalar-tensor theory possessing shift symmetry, $\phi \rightarrow \phi + c$, and reflection symmetry, $\phi \rightarrow -\phi$. The theory admits two arbitrary functions of $X := -(\partial\phi)^2/2$, and our solutions are constructed without specifying the concrete form of the two functions, implying that black hole solutions in specific scalar-tensor theories found in the literature can be extended to a more general class of theories with shift symmetry. Our solutions include a black hole in the presence of an effective cosmological constant, the Nariai spacetime, the Lifshitz black hole, and other nontrivial solutions, all of which exhibit nonconstant scalar-field profile.

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I. INTRODUCTION

Modifying general relativity has been one of the most highlighted fields in gravitational physics in recent years. Long distance modification of gravity has been studied extensively so as to explain the current accelerated expansion of the Universe (see, *e.g.*, Ref. [1] for a recent review). More theoretically, it is interesting to ask the simple question as to whether one can consistently modify general relativity to accommodate, *e.g.*, massive gravitons [2–4]. In many cases, modified theories of gravity can be described, at least effectively, by adding an extra scalar degree of freedom that participates in the dynamics of gravity. The most general Lagrangian composed of the metric $g_{\mu\nu}$ and a scalar ϕ and having second-order field equations will therefore be a powerful tool to study various aspects of modified gravity, and, interestingly, the theory was already constructed forty years ago by Horndeski [5]. Recently, the Horndeski theory was rediscovered [6, 7] and shown to be equivalent to the generalized galileon [8]. While considerable attention has been devoted to cosmological applications of the Horndeski theory, black holes in that theory have less been explored so far.

In the context of scalar-tensor modification of gravity, one of the central questions to address is whether or not black holes can have scalar hair. It has been proven by Hawking that a black hole cannot have scalar hair in the Brans-Dicke theory [9]. In the traditional scalar-tensor theory where ϕ is nonminimally coupled to gravity, the scalar-no-hair theorem was formulated in [10] (under the assumption of spherical symmetry), and a more general proof was provided recently in [11], while the no-hair the-

orem for a noncanonical scalar field, *i.e.*, k-essence, was given in [12]. It is then natural to ask how those results can be extended to theories whose Lagrangian contains second derivatives of ϕ . Such theories are motivated by the galileon [13], for which the equation of motion still remains of second order. For the galileon coupled to gravity, Hui and Nicolis have shown that static and spherically symmetric black holes cannot be surrounded by any nontrivial profile of the scalar field [14].

The key of the proof of Ref. [14] is shift symmetry of the scalar field, *i.e.*, symmetry under $\phi \rightarrow \phi + c$, where c is a constant, and the regularity of the square of the Noether current associated with this symmetry. Therefore, the same argument seems to hold for more general scalar-tensor theories with the same symmetry, though there is one specific example of shift symmetric theories signaling a potential loophole [15]. One can also circumvent the no-hair theorem [14] by abandoning the static configuration of ϕ and/or relaxing some asymptotic conditions on the metric and ϕ . In light of this latter loophole, exact black hole solutions with scalar hair have been found in the theory with nonminimal derivative coupling to the Einstein tensor, $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ [16–20].

The term $G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ has shift symmetry and reflection symmetry, $\phi \rightarrow -\phi$. The goal of this paper is to extend those previous works to go beyond this particular example, giving various exact black hole solutions with scalar hair in a subclass of the Horndeski theory possessing shift and reflection symmetries. The theory contains two *arbitrary* functions of $X := -(\partial\phi)^2/2$, and we will provide a variety of solutions without specifying the concrete form of those functions.

The paper is organized as follows. In the next section, we present the theory and the black hole ansatz considered in this paper. In Secs. III and IV, we give various exact solutions with scalar hair. The regularity of our solutions is discussed in Sec. V. Finally, we conclude in Sec. VI.

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II. SHIFT SYMMETRIC SCALAR-TENSOR THEORY AND BLACK HOLE ANSATZ

We consider a shift symmetric subclass of the Horndeski theory whose Lagrangian is given by

$$\mathcal{L} = G_2(X) + G_4(X)R + G_{4X} \left[(\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right], \quad (1)$$

where G_2 and G_4 are arbitrary functions of X , and $G_{4X} := \partial G_4 / \partial X$. The most general shift symmetric scalar-tensor theory with second-order field equations can accommodate two more arbitrary functions of X , often denoted as $G_3(X)$ and $G_5(X)$ in the literature. However, we restrict ourselves to the theory possessing the reflection symmetry as well, $\phi \rightarrow -\phi$, which forbids the G_3 and G_5 terms. We thus focus on the Lagrangian (1) describing the scalar-tensor theory with shift and reflection symmetries. Since

$$XR + (\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 = G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi, \quad (2)$$

up to a total divergence, we notice that the theory considered in Refs. [16–20] corresponds to the specific case with

$$G_2 = -2\Lambda_0 + 2\eta X, \quad G_4 = \zeta + \beta X, \quad (3)$$

where Λ_0, η, ζ , and β are constant parameters. (See Ref. [21] for numerical solutions of the theory (3) in the presence of an electromagnetic field.) In this paper, we go beyond the specific theory (3), leaving $G_2(X)$ and $G_4(X)$ arbitrary. As shown in the following, exact black hole solutions with a nontrivial configuration of ϕ can still be constructed.

Variation with respect to the metric yields the gravitational field equations,

$$\mathcal{E}_{\mu\nu} := \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L})}{\delta g^{\mu\nu}} = 0. \quad (4)$$

Shift symmetry of the theory allows us to write the scalar-field equation of motion as a current conservation equation,

$$\nabla_\mu J^\mu = 0, \quad (5)$$

where

$$\begin{aligned} J^\mu := & -G_{2X} \nabla^\mu \phi + 2G_{4X} G^{\mu\nu} \nabla_\nu \phi \\ & - G_{4XX} \left[(\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right] \nabla^\mu \phi \\ & - 2G_{4XX} (\Box\phi \nabla^\mu X - \nabla^\mu \nabla^\nu \phi \nabla_\nu X). \end{aligned} \quad (6)$$

The metric we are going to study is of the form

$$ds^2 = -h(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_K^2, \quad (7)$$

where $d\Omega_K^2$ is the metric of a unit two-dimensional sphere, plane, or hyperboloid for $K = +1, 0, -1$, respectively. We take the following t -dependent ansatz for the scalar field [17],

$$\phi(t, r) = qt + \psi(r), \quad q = \text{const}, \quad (8)$$

for which

$$X = \frac{1}{2} \left[\frac{q^2}{h} - f(\psi')^2 \right] \quad (9)$$

is, however, independent of t . Here and hereafter a prime denotes differentiation with respect to r . Since G_2 and G_4 are the functions of X only, it is more convenient to use X rather than ψ' when writing the field equations. Note in passing that, if $q \neq 0$, q can be chosen arbitrarily by rescaling the time coordinate: $q \rightarrow q/\alpha$, $t \rightarrow \alpha t$, and $h \rightarrow h/\alpha^2$ with constant α . This rescaling keeps the line element and X invariant.

III. $q \neq 0$ SOLUTIONS

We begin with the $q \neq 0$ case. An explicit calculation using the ansatz (8) shows that

$$\begin{aligned} J^r = & \frac{f\psi'}{r^2 h} \left\{ - (r^2 G_{2X} + 2K G_{4X}) h \right. \\ & \left. + 2[(G_{4X} + 2X G_{4XX})(rh)' - q^2 G_{4XX}] f \right\}, \end{aligned} \quad (10)$$

$$J^t = -\frac{qJ^r}{fh\psi'} - \frac{2q}{r} \left[G_{4X} \left(\frac{f}{h} \right)' + 2G_{4XX} \frac{f}{h} \right], \quad (11)$$

and $J^\Omega = 0$, where J^Ω stands for the other two components of the shift current J^μ . Since the (t, r) -component of the gravitational field equations can be written as

$$\mathcal{E}_{tr} = \frac{q}{f} J^r, \quad (12)$$

this equation and the scalar-field equation of motion (5) are satisfied by imposing

$$f = \frac{1}{2} \frac{(r^2 G_{2X} + 2K G_{4X}) h}{(G_{4X} + 2X G_{4XX})(rh)' - q^2 G_{4XX}}. \quad (13)$$

Now one can see from Eq. (11) that $J_\mu J^\mu = -h(J^t)^2$ does not diverge at the horizon as long as $(f/h)' < \infty$.

Using Eq. (13), we find

$$\begin{aligned} \mathcal{E}_{rr} = & -\frac{8X(G_{4X}^2 + G_4 G_{4XX})}{r^2(r^2 G_{2X} + 2K G_{4X})h} \\ & \times \left[(K - r^2 F)(rh)' - \frac{q^2}{2X} (K - r^2 \Lambda) \right], \end{aligned} \quad (14)$$

implying that

$$[K - r^2 F(X)](rh)' = \frac{q^2}{2X} [K - r^2 \Lambda(X)]. \quad (15)$$

Here we have defined the functions of X as

$$\Lambda(X) := -\frac{1}{2} \frac{G_2 G_{4XX} + G_{2X} G_{4X}}{G_{4X}^2 + G_4 G_{4XX}}, \quad (16)$$

$$\begin{aligned} F(X) &:= \frac{G_{2X} G_4 - G_2 G_{4X}}{4X(G_{4X}^2 + G_4 G_{4XX})} + \Lambda(X) \\ &= \frac{\partial_X(G_2 \mathcal{G})}{8X(G_{4X}^2 + G_4 G_{4XX})}, \end{aligned} \quad (17)$$

where

$$\mathcal{G}(X) := 2(G_4 - 2XG_{4X}). \quad (18)$$

Combining Eqs. (13) and (15), one notices the relation

$$f = \frac{2X}{q^2} [K - r^2 F(X)] h. \quad (19)$$

One can exploit the same set of equations, (13) and (15), to write the (t, t) -component of the gravitational field equations simply as

$$\mathcal{E}_{tt} = -\frac{2h^2}{q^2 r} \frac{d}{dr} [X \mathcal{G}^2 (K - r^2 F)] = 0. \quad (20)$$

This equation can be integrated to give

$$X \mathcal{G}^2(X) [K - r^2 F(X)] = C, \quad (21)$$

where C is an integration constant. Equation (21) determines $X = X(r)$ *algebraically*. Then, one can integrate Eq. (15) to determine $h(r)$, and finally obtains $f(r)$ from Eq. (13), or, more straightforwardly from Eq. (19), for given $G_2(X)$ and $G_4(X)$. The angular equations are written as $\mathcal{E}_{\Omega\Omega} = g_{\Omega\Omega} \mathcal{E}_{tt} r h' / 4h^2 = 0$ using the equations derived above, and hence are fulfilled automatically.

A. $F = 0$ solutions

A particularly simple solution of Eq. (21) is obtained if C is chosen in such a way that

$$C = C_0 := X_F \mathcal{G}^2(X_F) K, \quad (22)$$

where X_F is a *constant* satisfying

$$F(X_F) = 0. \quad (23)$$

In this case, $X = X_F$ trivially solves Eq. (21). Then, assuming that $K \neq 0$ one can integrate Eq. (15) immediately to get

$$h = -\frac{\mu}{r} + \frac{q^2}{2X_F K} \left(K - \frac{\Lambda(X_F)}{3} r^2 \right), \quad (24)$$

where μ is an integration constant. (For $K = 0$, Eq. (15) cannot be satisfied in general.) Substituting $F(X_F) = 0$ to Eq. (19), we arrive at

$$f = \frac{2X_F K}{q^2} h, \quad (25)$$

and therefore X_F must be such that $2X_F K > 0$ for the metric to be Lorentzian. We can then rescale the time coordinate to set $q^2 = 2X_F K$. The final form of the solution is thus

$$f = h = K - \frac{\Lambda(X_F)}{3} r^2 - \frac{\mu}{r}. \quad (26)$$

This is identical to the black hole metric in the presence of the (effective) cosmological constant $\Lambda(X_F)$, though it exhibits a nontrivial profile of $\phi(t, r)$. Interestingly, $\Lambda(X_F)$ can be nonzero even in the case where the true cosmological constant (which could be included in G_2) vanishes.

The radial profile of the scalar field for the above solution is given by integrating

$$(\psi')^2 = \frac{2X_F(1-h)}{fh}. \quad (27)$$

In order for ψ' to be real, we must require that $X_F(1-h) \geq 0$. This prohibits negative $\Lambda(X_F)$ (with $\mu > 0$). One would be concerned about the regularity of the scalar field at the horizon. As will be discussed in detail in Sec. V, ψ indeed diverges at the horizon. However, this seems not to be problematic because the action depends on the derivative of the scalar field rather than ϕ itself.

B. Nariai limit of the $F = 0$ solutions

Since the de Sitter-Schwarzschild metric solves the field equations as shown above, one can consider its extremal limit, *i.e.*, the Nariai spacetime [22]. In this Nariai limit, the spacetime between the black-hole and cosmological horizons can be described by

$$ds^2 = -\left(1 - \frac{z^2}{a^2}\right) d\tilde{t}^2 + \frac{dz^2}{(1 - z^2/a^2)} + a^2 d\Omega^2, \quad (28)$$

where $a := 1/\sqrt{\Lambda(X_F)}$ and the horizons are located at $z = \pm a$. The new coordinates \tilde{t} and z are related to the original ones by $\tilde{t} = \epsilon t$ and $z = \epsilon^{-1}(r - a)$, where the limit $\epsilon \rightarrow 0$ is taken keeping \tilde{t} and z fixed. In these coordinates, the scalar field is written as

$$\phi = \tilde{q} [\tilde{t} \pm a \operatorname{arctanh}(z/a)], \quad (29)$$

where $\tilde{q} = q/\epsilon$ is kept fixed in the $\epsilon \rightarrow 0$ limit. Even though ϕ shows a nontrivial profile and in particular $|\phi| \rightarrow \infty$ as $z \rightarrow \pm a$, we see that $X_F = 0$ and $J_\mu J^\mu = 0$ everywhere. We will discuss the regularity at the horizons in Sec. V.

C. $\mathcal{G} = 0$ solutions

Another class of $X = \text{const}$ solutions satisfying Eq. (20) can be obtained by choosing $X = X_{\mathcal{G}}$, where $\mathcal{G}(X_{\mathcal{G}}) = 0$. This gives the solution of the form

$$h = \frac{q^2}{2X_{\mathcal{G}}} \left[\frac{\Lambda(X_{\mathcal{G}})}{F(X_{\mathcal{G}})} + \left(1 - \frac{\Lambda(X_{\mathcal{G}})}{F(X_{\mathcal{G}})}\right) \mathcal{T}_\kappa(r) \right] - \frac{\mu}{r}, \quad (30)$$

where

$$\mathcal{T}_\kappa(r) := \begin{cases} \frac{\sqrt{\kappa}}{2r} \ln \left| \frac{r + \sqrt{\kappa}}{r - \sqrt{\kappa}} \right| & (\kappa > 0) \\ 0 & (\kappa = 0) \\ \frac{\arctan(r/\sqrt{-\kappa})}{r/\sqrt{-\kappa}} & (\kappa < 0) \end{cases}, \quad (31)$$

with $\kappa := K/F(X_G)$. We find $f = (2X_G F(X_G)/q^2)(\kappa - r^2)h$ by using Eq. (19), and then ψ by integrating Eq. (9).

Let us investigate the solution (30) closely for the three respective cases, $\kappa > 0$, $\kappa = 0$, and $\kappa < 0$. If $\kappa > 0$, $\Lambda/F \neq 1$, and $\mu > 0$, curvature singularities occur at $r = 0$ and $r = \sqrt{\kappa}$. However, those singularities can be hidden behind horizons.¹ This is indeed the case if $h(r)$ has two roots in the interval $0 < r < \sqrt{\kappa}$, which is realized for $X_G > 0$, $\Lambda > F > 0$, $K = +1$, with not too large $\mu > 0$. The geometry between the two zeros is similar to de Sitter-Schwarzschild. In the case of $\kappa > 0$, one can also consider the region $r > \sqrt{\kappa}$, with the singularity at $r = \sqrt{\kappa}$ being hidden inside the horizon. Such solutions with $h(r) > 0$ outside the horizon are obtained under either of the following conditions: $X_G > 0$, $F < \Lambda < 0$, $K = -1$ with sufficiently large μ ; $X_G > 0$, $\Lambda < F < 0$, $K = -1$ with arbitrary μ ; $X_G < 0$, $\Lambda < 0 < F$, $K = +1$ with arbitrary μ .

For $\kappa = 0$ and $\mu > 0$, f and h are positive outside the horizon provided that $\Lambda < 0$ and $X_G F < 0$. In this case we are allowed to set $q^2 = 2X_G F/\Lambda$ and the solution (30) reduces simply to

$$f = -\Lambda r^2 \left(1 - \frac{\mu}{r}\right), \quad h = 1 - \frac{\mu}{r}. \quad (32)$$

If $\kappa < 0$ and $\mu \neq 0$, a curvature singularity occurs only at $r = 0$. A solution for which $f, h > 0$ everywhere outside the horizon can be obtained if, for example, $\mu > 0$, $\Lambda < 0$, and $X_G F < 0$. In the case of $\mu = 0$ the solution is regular at the origin. For example, for $X_G > 0$, $F < 0$, and $\Lambda > 0$, f and h are positive only in a finite region around the origin, and the geometry is similar to the static region of the de Sitter spacetime.

Finally, the special case with $\mu > 0$, $X_G > 0$, $\Lambda(X_G) = F(X_G) > 0$, $K = +1$ (and hence $\kappa > 0$) reproduces the Schwarzschild black hole in an Einstein static universe discussed in Ref. [17].

D. Stealth Schwarzschild in the $G_2 = 0$ theory

Let us take a look at the theory with $G_2 = 0$. In this case, we have $F(X) = \Lambda(X) = 0$ for any G_4 , and hence Eq. (21) admits *only* the $X = \text{const}$ solutions. Taking

$q^2 = 2X$, from Eqs. (15) and (19) we obtain $f = h = 1 - \mu/r$ for $K = 1$. Therefore, a stealth Schwarzschild black hole solution can be obtained for more general G_4 than taken in [17] provided that $G_2 = 0$. For $K = 0$ ($K = -1$), Eq. (13) implies that $f = 0$ ($f < 0$), and therefore we do not have a sensible solution unless $K = 1$.

IV. $q = 0$ SOLUTIONS

For $q = 0$, Eqs. (13) and (15) reduce, respectively, to

$$f = \frac{r^2 G_{2X} + 2K G_{4X}}{G_{4X} + 2X G_{4XX}} \frac{h}{2(rh)'}, \quad (33)$$

and

$$K - r^2 F(X) = 0, \quad (34)$$

where $(rh)' \neq 0$ is assumed. The second equation determines $X = X(r)$ algebraically. The (t, t) -component of the field equations for $q = 0$ is given by

$$\mathcal{E}_{tt} = \frac{h}{r^2} \left[r^2 G_2 + 2K G_4 - \frac{1}{\mathcal{G}} \frac{d}{dr} (\mathcal{G}^2 r f) \right] = 0. \quad (35)$$

We obtain $f(r)$ by integrating Eq. (35). Then, Eq. (33) is used to fix $h(r)$.

Equation (34) implies that X must be dependent on r for $K \neq 0$. In this case, the explicit form of the metric is dependent on the concrete form of G_2 and G_4 . For example, in the theory (3), Eq. (34) yields a linear equation in X , which can be solved to give the solution presented in Refs. [16, 17, 19]. For $K = 0$, however, Eq. (34) forces X to be constant, $X = X_F$. Then, from Eqs. (33) and (35) we obtain

$$f = h = \frac{G_2(X_F)}{3\mathcal{G}(X_F)} r^2 - \frac{\mu}{r}, \quad (36)$$

where we used $\partial_X (G_2 \mathcal{G})|_{X=X_F} \propto F(X_F) = 0$. This is a planar anti-de Sitter black hole metric (for $G_2(X_F)/\mathcal{G}(X_F) > 0$). The profile of the scalar field is given by $(\psi')^2 = -2X_F/f$, implying that $\psi \sim \ln r \rightarrow \infty$ as $r \rightarrow \infty$, though its derivative is finite.

Let us finally consider the special class of $X = X_F = \text{const}$ solutions with $q = 0$, $K = 0$ satisfying

$$G_2(X_F) = \mathcal{G}(X_F) = 0. \quad (37)$$

(This condition is consistent with $F(X_F) = 0$.) In such theories, Eq. (35) is trivially fulfilled. One can then take *any* f and h provided that the two functions satisfy Eq. (33). For example, Eq. (33) admits the following solution:

$$f = r^2 \left(1 - \frac{\mu}{r^n}\right), \quad h = r^{b-1} \left(1 - \frac{\mu}{r^n}\right)^{b/n}, \quad (38)$$

where $b := -G_{2X}/\mathcal{G}_X|_{X=X_F}$ and n is arbitrary. A particular case $b = n (= 2z + 1)$ corresponds to the Lifshitz

¹ Note, however, that the propagation speeds of gravitational waves and a scalar-field fluctuation can be superluminal in the Horndeski theory.

black hole solution with the dynamical exponent z [20], though it was originally constructed in the specific theory (3). As another example, it is also easy to check that

$$f = br^2 \left(1 - \frac{\mu}{r}\right), \quad h = 1 - \frac{\mu}{r}, \quad (39)$$

fulfill Eq. (33). This solution gives the same geometry as obtained from Eq. (32).

V. REGULARITY AT THE HORIZON?

We are now in position to check the regularity of the scalar field at the horizon, $r = r_h$. Suppose that f and h are expanded near the horizon as

$$h = h_1(r - r_h) + \dots, \quad f = f_1(r - r_h) + \dots, \quad (40)$$

where f_1 and h_1 are constants. So far we have concentrated mainly on $X = \text{const}$ solutions. It turns out, however, that the regularity at the horizon does not depend on this property. One can determine X from Eq. (21) ($q \neq 0$) or Eq. (34) ($q = 0$), and X thus obtained may be constant or may be r -dependent. In any case, since those algebraic equations do not depend on the metric functions explicitly, nothing special happens to X at the horizon. This fact allows us to write

$$X = X_h + \mathcal{O}(r - r_h), \quad (41)$$

where the constant X_h is fixed by $X_h \mathcal{G}^2(X_h)[1 - r_h^2 F(X_h)] = C$ for $q \neq 0$ and $K - r_h^2 F(X_h) = 0$ for $q = 0$. Substituting Eqs. (40) and (41) to Eq. (11), one can see that $J_\mu J^\mu$ is also finite at the horizon. Thus, all the solutions we have found in this paper are regular in the sense that not only the spacetime but also the coordinate invariants constructed from the derivatives of scalar field, X and $J_\mu J^\mu$, are regular at the horizon.

Let us then examine the behavior of ϕ itself near the horizon, though the action depends on the scalar through $\partial_\mu \phi$ due to shift symmetry. From Eq. (9), it is found in the $q \neq 0$ case that

$$\begin{aligned} \psi' &= \pm \frac{q}{\sqrt{f_1 h_1}} \frac{1}{r - r_h} + \mathcal{O}((r - r_h)^0) \\ \Rightarrow \psi &= \pm \frac{q}{\sqrt{f_1 h_1}} \ln \left| \frac{r - r_h}{r_h} \right| + \mathcal{O}((r - r_h)^0). \end{aligned} \quad (42)$$

Therefore, ϕ diverges logarithmically at the horizon. As in [17], one may introduce the ingoing Eddington–Finkelstein coordinates (v, r) defined by $dv = dt + dr/\sqrt{f h}$ to write $\phi \simeq qv$ near the horizon, where the plus sign was chosen in Eq. (42). The scalar field appears to be regular in these coordinates. However, since the horizon is located at $v = \infty$, we notice that ϕ actually diverges at the horizon, implying that the divergence is not a coordinate artifact. Nevertheless, it should be emphasized again that the coordinate invariants constructed

from ϕ 's derivatives are regular, as we noted above. The argument here also applies to the $F = 0$ solutions in the Nariai limit by replacing r with z .

The situation is similar in some sense to the case of the so called BBMB solution which describes a black hole with conformally coupled scalar hair. In the BBMB solution, the scalar field diverges at the horizon while the metric and the curvature invariants remain finite there [23, 24]. Physical consequences of the divergence of ϕ would be clarified for instance by looking at the motion of a particle coupled to the scalar field and perturbations around the present background.

In the $q = 0$ case, it is easy to see $\phi \simeq 2[(-2X_h/f_1)(r - r_h)]^{1/2}$ near the horizon, and hence ϕ is regular at the horizon. As seen in the previous section, $\phi \rightarrow \ln r$ as $r \rightarrow \infty$ for the solutions in this class.

VI. CONCLUSIONS

In this paper, we have derived a variety of exact black hole solutions in a subclass of Horndeski's scalar-tensor theory having shift and reflection symmetries. Assuming the time-dependent ansatz for the scalar field [17], $\phi(t, r) = qt + \psi(r)$, we have obtained a solution describing a black hole in the presence of a cosmological constant and other nontrivial solutions without fixing the concrete form of the two arbitrary functions in the theory. This was made possible because the solutions we have explored have the key property: $X = \text{const}$. Our solutions circumvent the no-hair theorem because the scalar field itself is not necessarily static and it diverges at the horizon or at infinity. However, the action depends only on the derivatives of ϕ due to shift symmetry, and as a consequence the spacetime and the coordinate invariants constructed from the derivatives of the scalar field are regular at the horizon.

There are a lot of remaining issues to be addressed. One of the basic questions regarding a black hole solution is whether or not it is stable. It would therefore be interesting to study the stability issue by extending the black hole perturbation theory in Horndeski's scalar-tensor gravity formulated in the static background [25]. It would be also worth trying to generalize our solutions to rotating ones as has been done for a conformal scalar field [26], or to black holes with a realistic matter distribution as in the work [27] for typical modified gravity theories with a screening mechanism. We primarily focused on $X = \text{const}$ solutions in this paper, even though there could be many interesting solutions with r -dependent X . It would be useful to clarify the phase space of possible solutions and to discuss their phenomenological properties as in Refs. [28–31]. The theory we have studied admits superluminal propagations. It is crucial to study whether such superluminal propagations appear around the present black hole background in order to understand their causal structures and influence of singularities on the region exterior to the horizon.

Finally, we would like to remark that the Euclidean version of the theory (1) has been used in constructing a mechanism of emergent Lorentz signature [32]. Our black hole solutions may help to give an insight about such a novel scenario.

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